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AN ANALYSIS OF THE PERFORMANCE OF A
VERTICALLY ORIENTED, GAS LOADED,
VARIABLE CONDUCTANCE HEAT PIPE

by

Grey A. Glover, Sr.

March 1986

Thesis Advisor:

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An Analysis of the Performance of a
Vertically Oriented, Gas Loaded, Variable Conductance
Heat Pipe

by

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Lieutenant Commander, United States Navy
B.A., Youngstown State University, 1971

Submitted in partial fulfillment of the
requirements for the degree of

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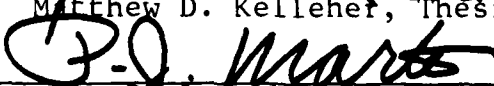
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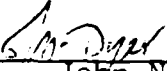
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ABSTRACT

An analysis of steady state operation of a vertically oriented, variable conductance heat pipe is presented. The effects of binary mass diffusion, axial pipe wall conduction, and gravitational effects, caused by the difference in molecular weight of the non-condensable gas and the working fluid are incorporated. Analytical expressions for the conservation of mass, momentum, and energy are combined along with equations of state to describe steady state operation of the heat pipe. These expressions are combined to form a system of three differential equations with three unknowns; working fluid vapor velocity, mass fraction, and temperature. These equations are nondimensionalized and an iteration scheme for numerical solution of the equations is presented.

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LIST OF SYMBOLS

A	=	area
A_c	=	vapor space cross sectional area
A_p	=	axial wick-wall cross sectional area
C	=	constant
D	=	binary mass diffusion coefficient
D_v	=	vapor space diameter
F	=	force
F_a	=	saturation pressure as a function of temperature
g	=	gravitational constant
h	=	convection heat transfer coefficient
h_{fg}	=	heat of vaporization
H_{fg}	=	heat of vaporization as a function of temperature
K	=	coefficient of thermal conductivity for wick-wall combination
L	=	length of heat pipe condenser
m	=	mass
\dot{m}	=	condensation mass flux
M	=	molecular weight
N	=	non-dimensional constant
P	=	pressure
P_s	=	perimeter of outside of pipe
P_v	=	perimeter of vapor space
Q	=	power (condenser-heat rejection rate)
q	=	axial conduction heat flux

R = universal gas constant
Sa = working fluid vapor saturation density as a function
of temperature
T = absolute temperature
V = velocity
x = axial distance

Greek Symbols

η = nondimensional distance
 θ = nondimensional temperature
 λ = nondimensional friction factor
 μ = viscosity
 ρ = density
 τ = viscous shear forces
 χ = mole fraction
 ω = mass fraction

Superscripts

* = nondimensional term

Subscripts

a = working fluid
b = noncondensable gas
c = condenser
e = evaporator
0 = initial value

I. INTRODUCTION

A. BACKGROUND

A heat pipe is a closed, evacuated chamber that has the inside walls lined with a wick structure, saturated with a volatile working fluid. The steady state operation involves vapor heat transfer and capillary action. A comprehensive coverage of the theory and operation of heat pipes can be found in Dunn and Reay [Ref. 1]. Briefly, the operation of a vertically oriented heat pipe will be discussed. The heat pipe has three sections: an evaporator section, an adiabatic section, and a condenser section. The heat pipe with the three sections marked is shown in Figure 1.1. The lower evaporator end of the heat pipe is heated, causing the working liquid in the wick to vaporize. This phase change causes the working fluid to absorb latent heat and the vapor pressure increases in the evaporator end. The working fluid is drawn away from the wick, causing the liquid pressure to decrease. The vapor moves to the cooler condenser where it is condensed and the working fluid gives up its latent heat. The condensate is deposited on the wick which causes the liquid level to increase thus increasing the liquid pressure. Capillary forces take the higher pressure liquid from the condenser to the lower pressure liquid end of the evaporator. The capillary forces are assisted by gravitational forces when the heat pipe is

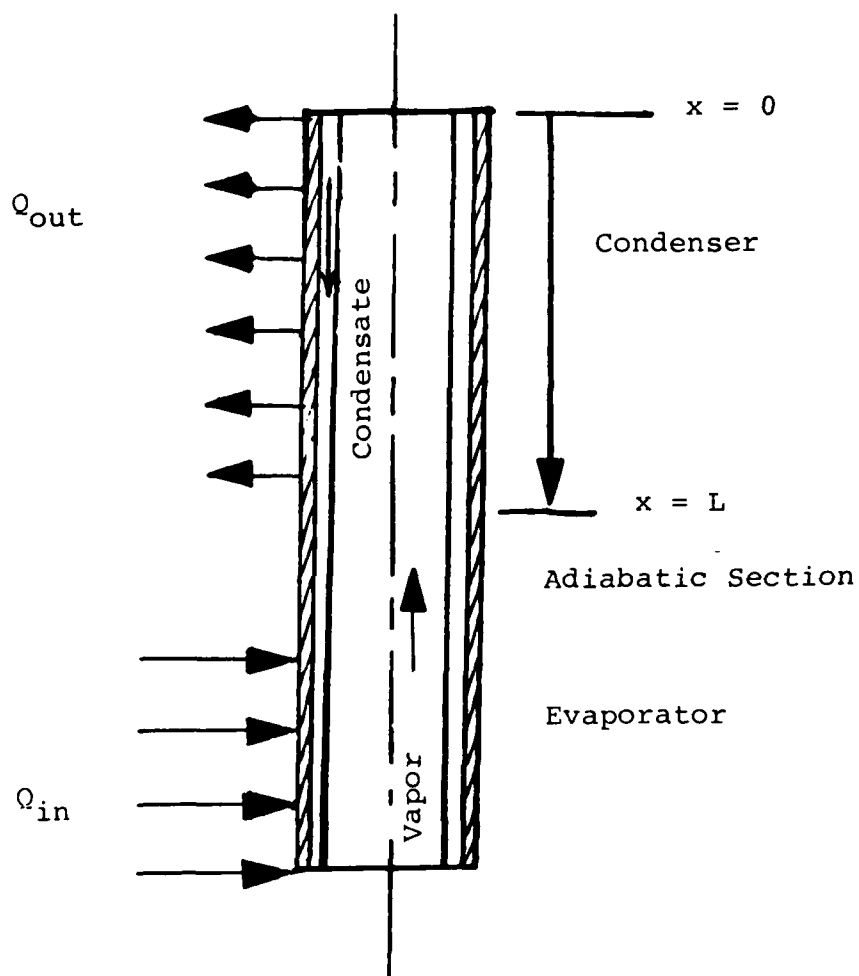


Figure 1.1. Heat Pipe

vertically oriented as shown in Figure 1.1. It is important to note that for heat pipes the evaporator position is not restricted, but since the experimental data and numerical model use a vertically oriented heat pipe, this orientation will be used throughout this thesis.

The great advantage of heat pipes is that the vaporization and condensation process allow for high heat transfer rates with small temperature differences. Two other advantages are that the basic heat pipe has no moving parts and can be used in many geometrical configurations. The heat pipe has limitations and again, a comprehensive presentation of these limitations can be found in Dunn and Reay [Ref. 1]. Briefly, the five heat pipe limitations are as follows:

- (a) The viscous limit is basically the low temperature limit. This would be the minimum temperature for the heat pipe to begin operation. Generally, the viscous limit is only important during start up of some heat pipes, i.e., liquid metal pipes.
- (b) The sonic limit describes the limit of the vapor flow velocity at the heat pipe evaporator exit. The resulting choked flow condition limits the axial heat flux capacity of the heat pipe.
- (c) The entrainment limitation is due to the removing of the liquid before it can return to the evaporator by vapor flowing at high velocity.
- (d) The wicking or capillary limit reflects the fact that each combination of wick and fluid has a maximum capillary head. As stated earlier, the capillary head must increase with the liquid and vapor pressure drops and can do so only up to the maximum capillary head.
- (e) The last limit is the boiling limit. If temperatures in the evaporator get too high the working fluid will begin to boil within the wick. This will disrupt liquid flow to the evaporator, causing the heat pipe to stop working.

These five heat pipe limitations vary in importance, depending on the temperature and corresponding axial heat flux operating point. Since at different temperatures one can encounter different heat pipe controlling limitations, it is important to have a basic understanding of all five limitations.

B. VARIABLE CONDUCTANCE HEAT PIPE

1. Basic Operation

Variable conductance heat pipes are described in detail in Marcus [Ref. 2]. As described in the background section, a change in the heat load will automatically change the operating temperature and pressure of the heat pipe, given a constant sink temperature. This is an undesirable quality for many situations where it is desired to have a stable source temperature under conditions of varying heat load or sink temperatures. An example of this case would be the cooling of sensitive electronic equipment. Basically, a noncondensable gas is introduced into the heat pipe along with the working fluid, making the heat pipe become a passively controlled device. The noncondensable gas occupies a certain portion of the condenser section, depending on operating conditions. Increasing operating temperature in turn increases the vapor pressure of the working fluid which compresses the noncondensable gas into a smaller volume. This provides a greater active condenser area, thus allowing a more stable source temperature than a conventional heat pipe. When the operating temperature decreases, working fluid vapor

pressure falls and the noncondensable gas expands, blocking a larger portion of the condenser. The result is a passively controlled variable condenser area, which in turn makes a more stable source temperature.

2. Variable Conductance Heat Pipe Theory

Marcus [Ref. 2] describes a flat front theory and a 'diffuse front theory' for variable conductance heat pipes. Flat front theory assumes the noncondensable gas is in the form of a plug with sharply defined vapor-gas interface which is perpendicular to the axis of the pipe. Since this model neglected the axial conduction in the heat pipe wall and diffusion of vapor into the stagnant gas plug, experimental results [Ref. 2] have shown this model to be inaccurate. The early model of flat front theory provided initial insight into the operating characteristics of gas loaded heat pipes. With the incorporation of binary diffusion between the working fluid vapor and the noncondensable gas and axial conduction in the pipe wall, Marcus' [Ref. 2] 'diffuse front theory' establishes a model of gas loaded heat pipes which provides better similarity between computer program and laboratory results.

Gravitational effects on the vapor-gas interface are not included in flat front theory or the 'diffuse front theory.' Gravitational effects can greatly distort the axial temperature profile when the working fluid and noncondensable gas have significantly different molecular weights. This fact is demonstrated by experimental results reported by Kelleher [Ref. 3].

Batts [Ref. 4] conducted variable conductance heat pipe experiments using working fluid/noncondensable gas combinations of methanol/helium and methanol/krypton. Kane [Ref. 5] developed an analytical model of variable conductance heat pipes which includes the effects of binary mass diffusion, axial pipe-wall conduction, and gravitational effects which exist at the vapor-gas interface region as shown by Batts [Ref. 4] experiments.

II. FORMULATION OF AN ANALYTICAL MODEL

A. PROBLEM STATEMENT

The purpose of this chapter is to formulate the equations describing the steady state operation of a vertically oriented, gas loaded, variable conductance heat pipe. As in Figure 1.1, the condenser is above the evaporator for this orientation. This analysis includes the effects of binary mass diffusion, axial pipe wall conduction, and gravitational effects due to the difference in molecular weight between the noncondensable gas and the working fluid.

B. ASSUMPTIONS

The following assumptions are used to develop the system of equations:

1. Heat, mass and momentum transport processes are treated as one-dimensional in the axial direction within the vapor space of a vertically oriented heat pipe. Kelleher [Ref. 3] shows that in any other orientation, gravitational effects cause highly three-dimensional spatial variations when the molecular weights of the working fluid and noncondensable gas differ substantially.
2. The noncondensable gas acts as a perfect gas. The mixture of noncondensable gas and working fluid are treated as thermodynamically independent substances.
3. Because of direct contact between the vapor and the liquid saturated wick, the working fluid vapor is at saturation conditions at all times.
4. The thermal resistance of the wick-wall structure, in the radial direction, is negligible in comparison with the thermal resistance due to convection outside the pipe.

C. ANALYSIS

Kane [Ref. 5] presents a proposed solution for this problem. The following is a significant change to his proposed solution with modifications being noted as they occur in this section. Variables with the subscript 'a' apply to the working fluid and variables with the subscript 'b' apply to the noncondensable gas.

Utilizing the assumption that the noncondensable gas is a perfect gas, the equation for noncondensable gas pressure is:

$$P_b = \rho_b \frac{R}{M_b} T \quad (\text{II.1})$$

The principles of mixtures of thermodynamically independent substances, which are found in Reynolds and Perkins [Ref. 6], are used to define the total density and the total pressure.

$$\rho = \rho_a + \rho_b \quad (\text{II.2})$$

$$P = P_a + P_b \quad (\text{II.3})$$

The definition of mass fraction is used to form the equation for the working fluid vapor mass fraction.

$$w_a = \frac{\rho_a}{\rho} \quad (\text{II.4})$$

The solution, utilizing the conservation of mass, is based on a differential element representing a section of the heat pipe condenser, Figure 2.1. The conservation of mass of species 'a' for the differential element yields:

$$\rho_a V_a A_c = (\rho_a V_a + \frac{d}{dx}(\rho_a V_a) dx) A_c + \dot{m}_a P_v dx .$$

This equation sets the mass of species 'a' into the control volume equal to the mass of species 'a' out, including a term to account for the removal of species 'a' by condensation. The process of condensation is accounted for by multiplying the condensation mass flux times the area, defined by the vapor space perimeter times the length, dx, of the differential element. After basic algebra, the equation for conservation of mass of species 'a' becomes:

$$0 = \dot{m}_a P_v + A_c \frac{d}{dx}(\rho_a V_a) .$$

By using Fick's law of binary diffusion [Ref.7], and solving for $\rho_a V_a$, the resulting formula is:

$$\rho_a V_a = \omega_a \rho V - \rho D \frac{d\omega_a}{dx} .$$

The final form for the conservation of mass for species 'a' becomes:

$$\dot{m}_a \frac{P_v}{A_c} + \frac{d}{dx}(\omega_a \rho V) - \frac{d}{dx}(\rho D \frac{d\omega_a}{dx}) = 0 . \quad (II.5)$$

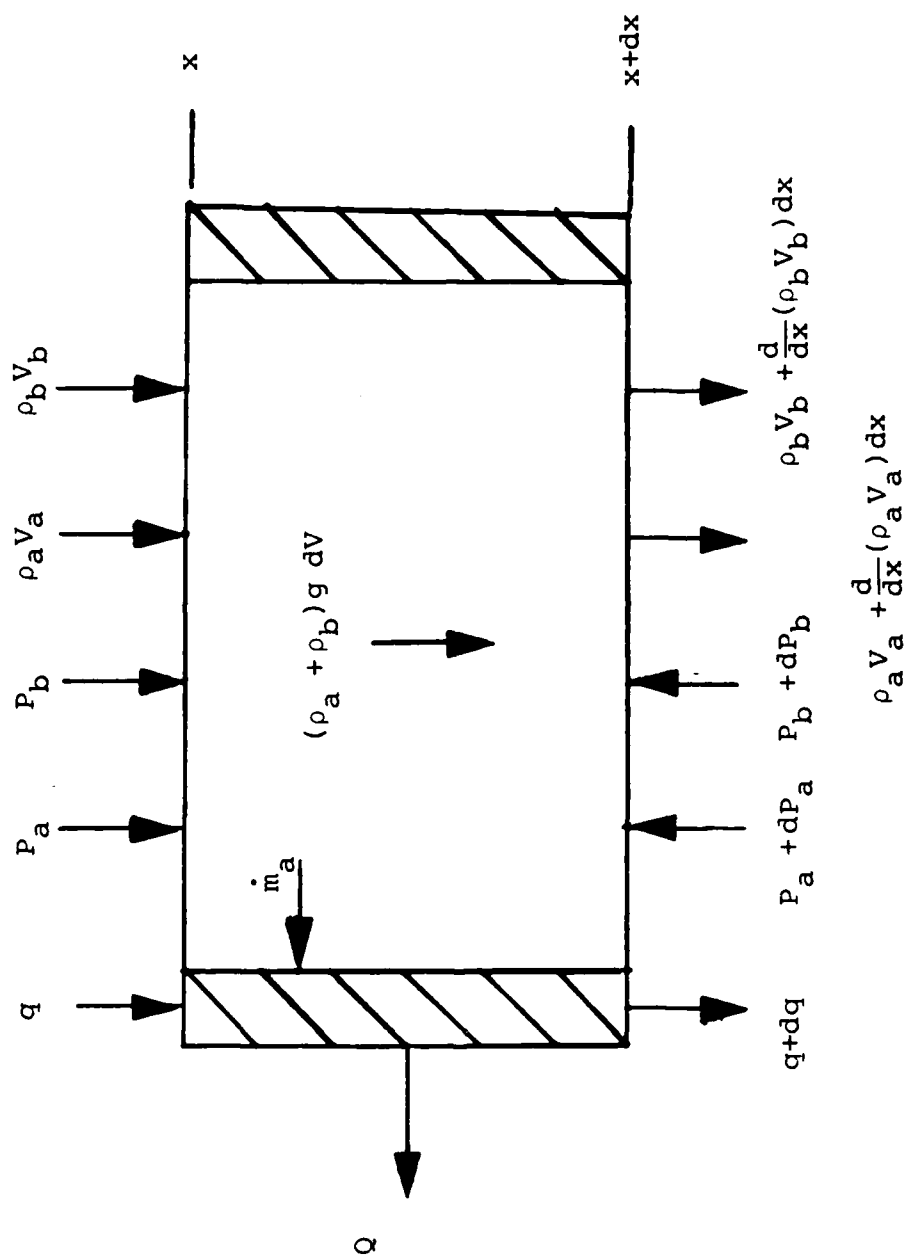


Figure 2.1. Differential Element for Conservation of Mass

Since species 'b' is a noncondensable gas, the condensation mass flux of species 'b' equals zero. The resulting equation for the conservation of mass for species 'b' is:

$$\frac{d}{dx}(\omega_b \rho V) - \frac{d}{dx}(\rho D \frac{d\omega_b}{dx}) = 0 .$$

The mass continuity of the mixture is derived by adding the conservation of mass of species 'a' with the conservation of mass of species 'b'.

$$\dot{m}_a \frac{P_v}{A_c} + \frac{d}{dx}(\rho V(\omega_a + \omega_b)) - \frac{d}{dx}(\rho D(\frac{d\omega_a}{dx} + \frac{d\omega_b}{dx})) = 0 .$$

Because $\omega_a + \omega_b$ is equal to one, and $\frac{d}{dx}(\omega_a + \omega_b) = \frac{d}{dx}(1) = 0$, the final result of the mass continuity of the mixture is:

$$\dot{m}_a \frac{P_v}{A_c} + \frac{d}{dx}(\rho V) = 0 . \quad (II.6)$$

Figure 2.2 is a differential element of the vapor space in the condenser and is used to derive the conservation of linear momentum equation for the mixture of noncondensable gas and working fluid vapor. The conservation of linear momentum for the mixture of noncondensable gas and working fluid vapor is derived by setting the summation of external forces acting on the control volume in the x direction equal to the net efflux of linear momentum.

$$\sum \vec{F}_x = \int_{CS,x} \vec{V}(\rho \vec{V} d\vec{A})$$

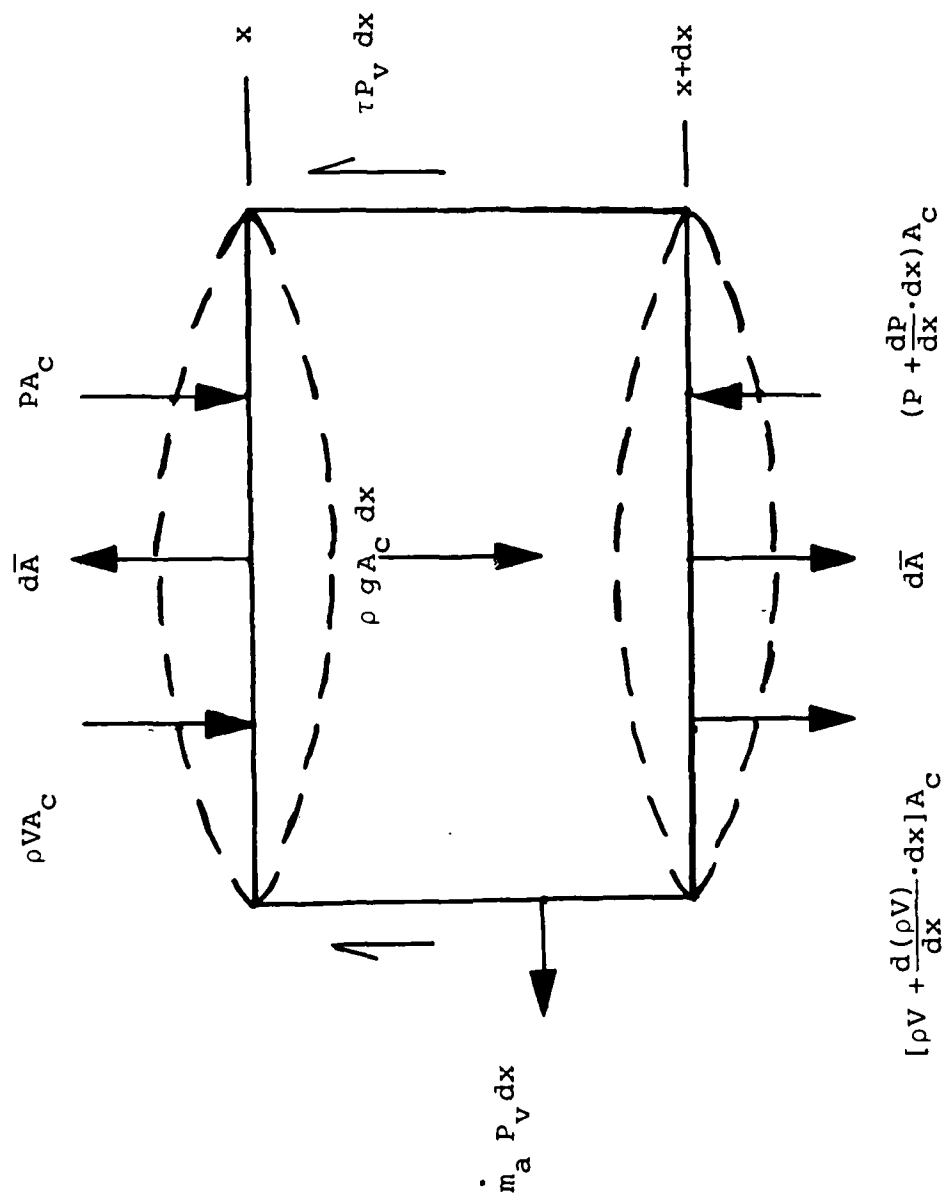


Figure 2.2. Differential Element for Conservation of Momentum

The summation of external forces acting in the x direction are:

$$\sum \vec{F}_x = \rho g A_c dx - \tau P_v dx - dp A_c .$$

These terms represent the force due to gravity acting on the control volume minus the forces due to wall shear stress and pressure acting on the control volume. The wall shear stress can be calculated by:

$$\tau = \lambda \frac{\rho}{2} V^2$$

where λ is a dimensionless friction factor. If developed laminar flow is assumed the Darcy friction factor, $\lambda = \frac{16}{Re}$, could be used with the Reynolds number defined as

$$Re = \frac{\rho V D_v}{\mu}$$

The equation for wall shear stress then becomes:

$$\tau = \frac{8\mu}{D_v} V$$

where μ is the viscosity of methanol vapor, V is the velocity of the vapor and D_v is the diameter of the vapor space. The terms to represent the net efflux of linear momentum are:

$$\begin{aligned} \int_{CS,x} \vec{V}(\rho \vec{v} d\vec{A}) &= V_a \dot{m}_a P_v dx + (\rho v + d(\rho v)(v+dv)) A_c - \rho V^2 A_c \\ &= V_a \dot{m}_a P_v dx + A_c \rho v dv + A_c v d(\rho v) . \end{aligned}$$

By setting the summation of external forces in the x direction equal to the net efflux of linear momentum and combining like terms, the result is:

$$\frac{dP}{dx} = \rho g - \frac{d}{dx}(\rho v^2) - \frac{P_v}{A_c} V \frac{\dot{m}_a}{\omega_a} - \frac{P_v}{A_c} \frac{8\mu v}{D_v} \quad (\text{II.7})$$

The above equation represents conservation of linear momentum for the mixture of noncondensable gas and working fluid vapor. Kane [Ref. 5] assumes zero velocity for the noncondensable gas and derives an equation for conservation of linear momentum of species 'b.' The species 'b' conservation of linear momentum equation is not necessary to derive the model in this analysis. Since the conservation of linear momentum for the mixture of the noncondensable gas and working fluid vapor is the more consistent approach, Equation (II.7) is used throughout this thesis.

Figure 2.3 is a differential element of the condenser wall and represents the wick and pipe wall combination as one element. The wick-wall combination is treated like a fin and is used to derive the equation for the conservation of energy. Heat transfer is represented by the latent heat deposited on the 'fin' by condensation of the working fluid vapor plus the heat transfer from Fourier's law of conduction entering the 'fin.'

$$Q_{in} = \dot{m}_a h_{fg} P_v dx + Q_c$$

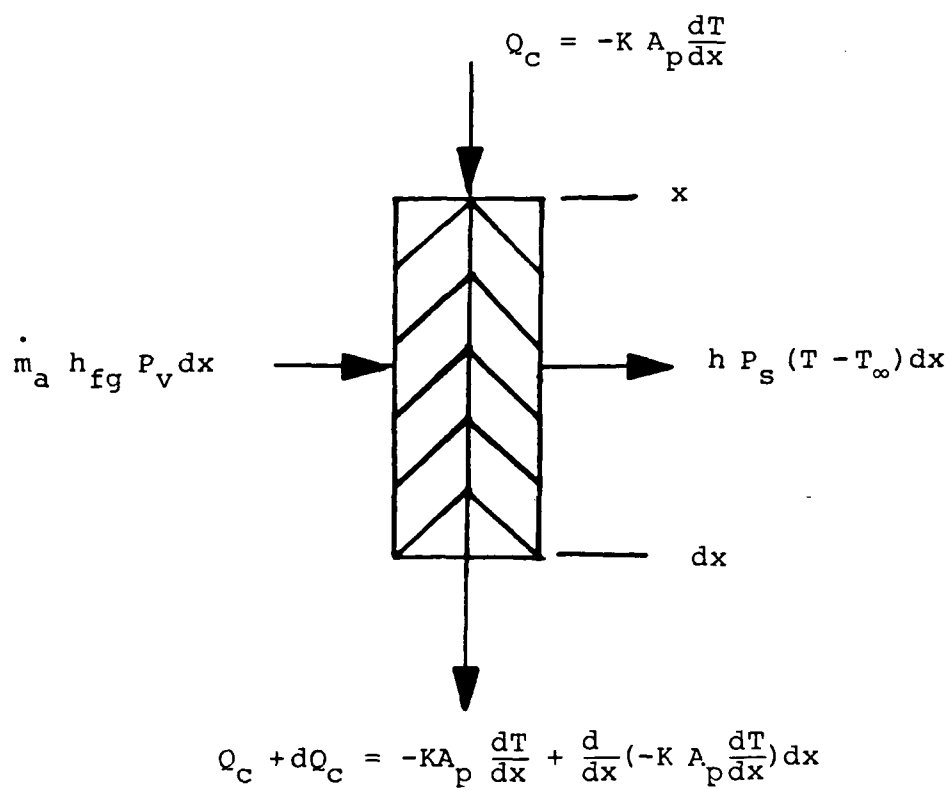


Figure 2.3. Differential Element for Energy Balance

Heat transfer out is represented by Fourier's law of conduction leaving the 'fin' plus the energy which is convected away from the fin.

$$Q_{out} = Q_c + dQ_c + h P_s (T - T_\infty) dx$$

The equation for conservation of energy is derived by setting heat transfer in equal to heat transfer out of the 'fin.'

$$\frac{d^2 T}{dx^2} - \frac{h P_s}{k A_p} (T - T_\infty) + \frac{h_{fg} P_v}{k A_p} \dot{m}_a = 0 \quad (II.8)$$

The experimental data from Batts [Ref. 4] is used to correlate this analysis. The working fluid is methanol, CH_3OH . The working fluid vapor in the heat pipe is in direct contact with the working fluid liquid in the wick and is therefore considered to be a saturated vapor at the local temperature of the wick-wall combination. Eubank [Ref. 8] tabulates data for the saturation properties of methanol. These tables are used to generate functions of local temperature for partial pressure, partial density and heat of vaporization of saturated methanol vapor.

(a) The partial pressure of saturated methanol vapor is a function, F_a , of local temperature.

$$P_a = F_a(T) \quad (II.9)$$

(b) The partial density of saturated working fluid vapor is a function, S_a , of local temperature.

$$\rho_a = S_a(T) \quad (\text{II.10})$$

(c) The working fluid heat of vaporization is a function, H_{fg} , of local temperature.

$$h_{fg} = H_{fg}(T) \quad (\text{II.11})$$

In summary, the eleven equations which are stated in this chapter form the foundation for the analysis of a vertically oriented, variable conductance heat pipe. The remaining chapters derive a numerical model and iteration scheme based on the eleven equations from this chapter.

III. NUMERICAL MODEL FORMULATION

A. VARIABLE ELIMINATION

The system of eleven equations can ultimately be reduced to a system of three equations by eliminating variables. Equation (II.9) is eliminated by using the equation of state, $f_a(T)$, for P_a . The partial density of the saturated working fluid vapor, ρ_a (Equation II.10) is eliminated by using $S_a(T)$. $H_{fg}(T)$, Equation (II.11), is used to eliminate h_{fg} . The noncondensable gas pressure, P_b , is eliminated by using the perfect gas equation of state (Equation II.1). The treatment of working fluid vapor and noncondensable gas as thermodynamically independent substances gives total pressure as the sum of partial pressures and total density as the sum of partial densities. Total pressure is eliminated by using the sum of partial pressures, Equation (II.3). The noncondensable gas density is eliminated by solving Equation (II.2) for ρ_b and replacing ρ_b with $\rho - \rho_a$. The definition for mass fraction of the working fluid vapor, Equation (II.4), is used to replace total density.

After making the preceding seven substitutions, four equations with four unknowns are left. The four remaining dependent variables which are functions of axial position are:

- (a) \dot{m}_a , working fluid condensation flux [lbm/ft² sec]
- (b) V , velocity of the mixture [ft/sec]
- (c) ω_a , working fluid mass fraction [$0 \leq \omega_a \leq 1$]
- (d) T , temperature [$^{\circ}\text{R}$].

The four remaining equations are:

$$(a) \quad \dot{m}_a \frac{P_v}{A_c} + \frac{d}{dx} \left(\frac{Sa}{\omega_a} v \right) = 0 \quad (III.1)$$

$$(b) \quad \dot{m}_a \frac{P_v}{A_c} + \frac{d}{dx} (Sa v) - \frac{d}{dx} \left(\frac{Sa D}{\omega_a} \frac{d\omega_a}{dx} \right) = 0 \quad (III.2)$$

$$(c) \quad \frac{dFa}{dx} + \frac{d}{dx} \left(Sa \left(\frac{1}{\omega_a} - 1 \right) \frac{R}{M_b} T \right) \\ = \left(\frac{Sa}{\omega_a} g \right) - \frac{d}{dx} \left(\frac{Sa}{\omega_a} v^2 \right) - \frac{P_v}{A_c} v \frac{\dot{m}_a}{\omega_a} - \frac{P_v}{A_c} \frac{8\mu}{D_v} v \quad (III.3)$$

$$(d) \quad \frac{d^2 T}{dx^2} - \frac{P_s}{KA_p} h (T - T_\infty) + \frac{H_{fg} P_v}{KA_p} \dot{m}_a = 0 \quad (III.4)$$

Furthermore, Equation (III.1) can be eliminated by solving this equation for working fluid condensation flux, \dot{m}_a , and substituting that into the three remaining equations.

The resulting three equations with dependent variables of velocity, working fluid mass fraction, and temperature are:

$$(a) \quad -\frac{d}{dx} \left(\frac{Sa}{\omega_a} v \right) + \frac{d}{dx} (Sa v) - \frac{d}{dx} \left(\frac{Sa}{\omega_a} D \frac{d\omega_a}{dx} \right) = 0 \quad (III.5)$$

$$(b) \quad \frac{dFa}{dx} + \frac{d}{dx} \left(Sa \left(\frac{1}{\omega_a} - 1 \right) \frac{R}{M_b} T \right) \\ = \left(\frac{Sa}{\omega_a} g \right) - \frac{d}{dx} \left(\frac{Sa}{\omega_a} v^2 \right) + \frac{v}{\omega_a} \frac{d}{dx} \left(\frac{Sa}{\omega_a} v \right) - \frac{P_v 8\mu}{A_c D_v} v \quad (III.6)$$

$$(c) \quad \frac{d^2 T}{dx^2} - \frac{P_s}{KA_p} h (T - T_\infty) - \frac{H_{fg} A_c}{KA_p} \frac{d}{dx} \left(\frac{Sa}{\omega_a} V \right) = 0 \quad (III.7)$$

This set of three equations and three unknowns describe the operation of a vertically oriented, gas loaded, variable conductance heat pipe.

B. BOUNDARY CONDITIONS

These equations describe the thermodynamics of the heat pipe condenser. The independent variable, the distance along the heat pipe, is measured with the origin at the condenser end farthest from the evaporator. The total condenser length is defined as L . Based on the system of three equations, two boundary conditions are required for temperature and mass fraction and one boundary condition is required for velocity.

From experimental data, the temperature at the far end of the condenser and in the evaporator are known. The boundary conditions for the dependent variable of temperature are:

when $x = 0$, $T = T_c$, where T_c equals the temperature at the far end of the condenser.

when $x = L$, $T = T_e$, where T_e equals the temperature at the end of the condenser closest to the evaporator.

The highest velocity of the working fluid vapor is reached in the evaporator exit. This velocity can be calculated from the known heat input to the heat pipe. The vapor velocity in the evaporator, V_e , is equal to the heat transfer rate into

the evaporator divided by the product of local density of the working fluid vapor, cross sectional area of the vapor space, and the local heat of vaporization of the working fluid. Due to the physical boundary at the end of the pipe, where $x = 0$, the velocity at the farthest end of the condenser away from the evaporator is equal to zero. The boundary conditions for the dependent variable of velocity are:

when $x = 0$, $V = 0.0$ and

$$\text{when } x = L, \quad V = V_e = -\frac{Q_e}{\rho_{a_e} A_c h_{fg_e}}.$$

Based on the system of three equations, only one of the above boundary conditions is required. The boundary condition at the origin is used for the iteration scheme and V_e is used to normalize velocity.

The temperature and pressure at the farthest end of the condenser from the evaporator are known from laboratory data. Assuming thermodynamically independent substances, the total pressure is the sum of partial pressures and the total density is the sum of partial densities. The saturation pressure of the working fluid vapor, P_a , could be determined from tabulated data by entering the saturation pressure table with condenser temperature. An example of this table can be found in Eubank [Ref. 8] for methanol saturated vapor. The noncondensable gas pressure, P_b , can be calculated by subtracting the working

fluid vapor pressure from the total pressure. With this data, the initial mass fractions can be established by using the following procedure from Reynolds and Perkins [Ref. 6]:

$$\chi_a = \frac{P_a}{P_{\text{total}}}$$

$$\chi_b = \frac{P_b}{P_{\text{total}}}$$

$$\chi_a M_a = \frac{L_{bm}}{L_{bmole \text{ mixture}}} \quad \text{for species 'a'}$$

$$\chi_b M_b = \frac{L_{bm}}{L_{bmole \text{ mixture}}} \quad \text{for species 'b'}$$

$$\omega_{a0} = \frac{\chi_a M_a}{\chi_a M_a + \chi_b M_b}$$

$$\omega_{b0} = \frac{\chi_b M_b}{\chi_a M_a + \chi_b M_b}$$

At the end of the condenser closest to the evaporator, it is assumed that the concentration of noncondensable gas is zero so that at this point ω_a is equal to one. The boundary conditions for the dependent variable of mass fraction for the working fluid vapor are:

when $x = 0$, $\omega_a = \omega_{a0}$, and

when $x = L$, $\omega_a = 1.0$.

C. EQUATION NONDIMENSIONALIZATION

To generalize the equations and facilitate their solution it is convenient to use dimensionless variables defined as follows.

(a) The independent variable, condenser length, is defined by:

$$\eta = \frac{x}{L}$$

where x is the distance along the heat pipe, measured from the condenser end farthest from the evaporator, and L is the total condenser length.

(b) A dimensionless temperature can be defined by:

$$\theta = \frac{T - T_{\infty}}{T_e - T_{\infty}}$$

where T is the local temperature, T_e is the evaporator temperature, and T_{∞} is the ambient temperature. In this analysis, the first and second derivative of temperature with respect to distance are required.

$$\frac{dT}{dx} = \frac{dT}{d\theta} \frac{d\theta}{d\eta} \frac{d\eta}{dx} = \frac{T_e - T_{\infty}}{L} \frac{d\theta}{d\eta}$$

and

$$\begin{aligned} \frac{d^2T}{dx^2} &= \frac{d}{dx} \left(\frac{dT}{dx} \right) = \frac{d}{d\eta} \left(\frac{T_e - T_{\infty}}{L} \frac{d\theta}{d\eta} \right) \frac{d\eta}{dx} \\ &= \frac{T_e - T_{\infty}}{L^2} \frac{d^2\theta}{d\eta^2} \end{aligned}$$

(c) The second dependent variable, working fluid mass fraction, is dimensionless by definition and its numerical value is greater than or equal to zero and less than or equal to one. The first derivative of working fluid mass fraction with respect to distance is:

$$\frac{d\omega_a}{d\eta} = \frac{d\omega_a}{d\eta} \frac{d\eta}{dx} = \frac{1}{L} \frac{d\omega_a}{d\eta} .$$

(d) Velocity of the working fluid vapor is made dimensionless by normalizing local vapor velocity with the evaporator vapor velocity, V_e . The nondimensional velocity and the first derivative of velocity with respect to distance are then defined by:

$$v^* = \frac{v}{V_e}$$

$$\frac{dv}{dx} = \frac{dv}{dv^*} \frac{dv^*}{d\eta} \frac{d\eta}{dx} = \frac{V_e}{L} \frac{dv^*}{d\eta} .$$

(e) In order to complete nondimensionalizing process, the following other dimensionless variables are used.

(i) The working fluid vapor density is normalized with respect to the evaporator vapor density.

$$Sa^* = \frac{Sa}{Sa_e}$$

(ii) The working fluid vapor pressure is compared to the evaporator vapor pressure.

$$Fa^* = \frac{Fa}{Fa_e}$$

(iii) The working fluid heat of vaporization is normalized with respect to the condenser working fluid vapor heat of condensation.

$$H_{fg}^* = \frac{H_{fg}}{H_{fg,c}}$$

(iv) The working fluid vapor viscosity is normalized with respect to the evaporator vapor viscosity.

$$\mu^* = \frac{\mu}{\mu_e}$$

After substitution of nondimensional terms, the three equations which describe operation of a vertical variable conductance heat pipe become:

$$(a) \quad - \frac{d}{d\eta} \left(\frac{Sa^* \cdot V^*}{\omega_a} \right) + \frac{d}{d\eta} (Sa^* V^*) - \frac{D}{LV_e} \left(\frac{d}{d\eta} \frac{Sa^*}{\omega_a} \frac{d\omega_a}{d\eta} \right) = 0 \quad (III.8)$$

The dimensionless coefficient from Equation (III.8) is

$$N1 = \frac{D}{LV_e}.$$

$$\begin{aligned}
(b) \quad & \frac{Fa_e}{Sa_e V_e^2} \frac{dFa^*}{d\eta} + \frac{R(T_e - T_\infty)}{M_b V_e^2} \left(\frac{d}{d\eta} (Sa^* (\frac{1}{\omega_a} - 1) (\theta + \frac{T_\infty}{T_e - T_\infty})) \right) \\
& = \frac{Lg}{V_e^2} \frac{Sa^*}{\omega_a} - \frac{d}{d\eta} \left(\frac{Sa^* V^{*2}}{\omega_a} \right) + V^* \frac{d}{d\eta} \left(\frac{Sa^* V^*}{\omega_a} \right) \\
& \quad - \frac{P_v}{A_c} \frac{8 \mu_e L}{D_v Sa_e V_e} \mu^* V^* \quad (III.9)
\end{aligned}$$

The dimensionless coefficients from Equation (III.9) are

$$\begin{aligned}
N2 &= \frac{Fa_e}{Sa_e V_e^2}, \quad N3 = \frac{R(T_e - T_\infty)}{M_b V_e^2}, \quad N4 = \frac{T_\infty}{T_e - T_\infty}, \\
N5 &= \frac{Lg}{V_e^2}, \quad \text{and} \quad N6 = \frac{P_v}{A_c} \frac{8 \mu_e L}{D_v Sa_e V_e}.
\end{aligned}$$

$$\begin{aligned}
(c) \quad & \frac{d^2 \theta}{d\eta^2} - \frac{P_s h L^2}{K A_p} \theta - \frac{H_{fg,c} A_c Sa_e V_e L}{K A_p (T_e - T_\infty)} H_{fg}^* \\
& \times \frac{d}{d\eta} \left(\frac{Sa^* V^*}{\omega_a} \right) = 0 \quad (III.10)
\end{aligned}$$

The two nondimensional terms of interest from Equation (III.10) are

$$N7 = \frac{P_s h L^2}{K A_p} \quad \text{and} \quad N8 = \frac{H_{fg,c} A_c Sa_e V_e L}{K A_p (T_e - T_\infty)}.$$

Numerical evaluation of the dimensionless coefficients is required to determine the relative significance of each term.

Two experimental runs from Batts [Ref. 4] are used to evaluate the dimensionless coefficients. A low powered run ($Q = 22.9$ watts) with methanol as the working fluid, and krypton as the noncondensable gas, is chosen. This run demonstrates an example of the heavier molecular weight noncondensable gas diffusing through a large portion of the condenser. A second higher powered run ($Q = 70$ watts) with methanol as the working fluid, and helium as the noncondensable gas, is chosen because it demonstrates an example of the lighter molecular weight noncondensable gas occupying a small portion of the condenser. The convection heat transfer coefficient for the heat pipe is set equal to $1.0 \frac{\text{Btu}}{\text{hr.ft.}^2\text{°R}}$, representing free convection. A system thermal conductivity of approximately $4.56 \frac{\text{Btu}}{\text{hr.ft.}^2\text{°R}}$ results from a parallel circuit combination of conductive thermal resistances for the stainless steel pipe wall and methanol filled stainless steel wire mesh wick. Kane [Ref. 5] calculated representative binary mass diffusion coefficients of $0.4 \text{ ft}^2/\text{hr}$ for krypton-methanol and $1.9 \text{ ft}^2/\text{hr}$ for helium-methanol. These values for binary mass diffusion are also used in this analysis. The experimental values that are used to calculate the dimensionless constants are listed in Table I. The values of the dimensionless constants for the two experimental runs are listed in Table II.

Based on the tabulated numerical results, Equation (III.9) is modified to:

TABLE I
NUMERICAL DATA

Constants for both runs:

$R = 1545.33 \text{ ft Lbf/Lbmole } ^\circ\text{R}$	Gas constant
$g = 32.2 \text{ ft/sec}^2$	Acceleration of gravity
$L = 3.5 \text{ ft}$	Length of condenser
$A_c = .0199 \text{ ft}^2$	Cross sectional area of vapor space
$A_p = .001917 \text{ ft}^2$	Axial cross sectional area of wick-wall combination
$D_v = .15917 \text{ ft}$	Diameter of vapor space
$P_s = .524 \text{ ft}$	Periphery of outside of pipe
$P_v = .500 \text{ ft}$	Periphery of vapor space
$h = 1.0 \text{ Btu/hr ft}^2 ^\circ\text{R}$	Convection heat transfer coefficient
$K = 4.5585 \text{ Btu/hr ft } ^\circ\text{R}$	Thermal conductivity
Low powered krypton/methanol	
$Q = 22.9 \text{ watts} = 78.1348 \text{ Btu/hr}$	
$T_e = 592 ^\circ\text{R}$	
$T_c = 543 ^\circ\text{R}$	
$T_\infty = 536 ^\circ\text{R}$	
$D = 0.4 \text{ ft}^2/\text{hr}$	
$M_b = 83.8 \text{ Lbm/Lbmole}$	

TABLE I (CONTINUED)

$$V_e = .041339 \text{ ft/sec}$$

$$Fa_e = 10.24 \text{ Lbf/in}^2$$

$$H_{fg,c} = 500.21 \text{ Btu/Lbm}$$

$$Sa_e = 0.054871 \text{ Lbm/ft}^3$$

$$\mu_e = 224.502 \times 10^{-9} \text{ Lbf sec/ft}^2$$

High powered helium/methanol

$$Q = 70 \text{ watts} = 238.84 \text{ Btu/hr}$$

$$T_e = 640^\circ\text{R}$$

$$T_c = 540^\circ\text{R}$$

$$T_\infty = 536^\circ\text{R}$$

$$D = 1.9 \text{ ft}^2/\text{hr}$$

$$M_b = 4.003 \text{ Lbm/Lbmole}$$

$$V_e = .050485 \text{ ft/sec}$$

$$Fa_e = 28.52 \text{ Lbf/in}^2$$

$$H_{fg,c} = 501.11 \text{ Btu/Lbm}$$

$$Sa_e = 0.14446 \text{ Lbm/ft}^3$$

$$\mu_e = 243.993 \times 10^{-9} \text{ Lbf sec/ft}^2$$

TABLE II
DIMENSIONLESS CONSTANTS

Low Powered Krypton/Methanol

High Powered Helium/Methanol

$$N1 = \frac{D}{LV_e} = 7.679 \times 10^{-4}$$

$$N1 = \frac{D}{LV_e} = 2.9869 \times 10^{-3}$$

$$N2 = \frac{Fa_e}{Sa_e V_e^2} = 506.355 \times 10^6$$

$$N2 = \frac{Fa_e}{Sa_e V_e^2} = 359.166 \times 10^6$$

$$N3 = \frac{R(T_e - T_\infty)}{M_b V_e^2} = 19.4581 \times 10^6$$

$$N3 = \frac{R(T_e - T_\infty)}{M_b V_e^2} = 507.224 \times 10^6$$

$$N4 = \frac{T_\infty}{T_e - T_\infty} = 9.571$$

$$N4 = \frac{T_\infty}{T_e - T_\infty} = 5.154$$

$$N5 = \frac{Lg}{V_e^2} = 65948.4$$

$$N5 = \frac{Lg}{V_e^2} = 44218.0$$

$$N6 = \frac{P_v 8 \mu_e L}{A_c D_v Sa_e V_e} = 14.086$$

$$N6 = \frac{P_v 8 \mu_e L}{A_c D_v Sa_e V_e} = 4.76143$$

$$N7 = \frac{P_s h L^2}{K A_p} = 734.553$$

$$N7 = \frac{P_s h L^2}{K A_p} = 734.553$$

$$N8 = \frac{H_{fg,c} A_c Sa_e V_e L}{K A_p (T_e - T_\infty)}$$

$$N8 = \frac{H_{fg,c} A_c Sa_e V_e L}{K A_p (T_e - T_\infty)}$$

$$= 581.362$$

$$= 1008.30$$

$$\begin{aligned} \frac{Fa_e}{Sa_e V_e^2} \frac{dFa^*}{d\eta} + \frac{R(T_e - T_\infty)}{M_b V_e^2} \left[\frac{d}{d\eta} \left(Sa^* \left(\frac{1}{\omega_a} - 1 \right) \left(\theta + \frac{T_\infty}{T_e - T_\infty} \right) \right) \right] \\ = \frac{Lg}{V_e^2} \frac{Sa^*}{\omega_a} . \end{aligned} \quad (III.11)$$

The viscous term and the two inertia terms are neglected because their order of magnitude is 10^1 compared to 10^3 and 10^6 for the remaining terms. The nondimensional equations (III.8), (III.10), and (III.11) are used to develop the analytical model of the vertically oriented, variable conductance heat pipe.

D. FORMULATION OF THE NUMERICAL MODEL

Equation (III.8) is integrated and is solved for the first derivative of the working fluid mass fraction. Integrating both sides of Equation (III.8) gives:

$$- \frac{Sa^* V^*}{\omega_a} + Sa^* V^* - Nl \frac{Sa^*}{\omega_a} \frac{d\omega_a}{d\eta} = C . \quad (III.12)$$

The lower endpoint is used ($\eta = 0$) to evaluate the constant of integration. When $\eta = 0$, V^* at the end of the pipe equals zero. The constant of integration then becomes:

$$C = -Nl \frac{\rho_0}{Sa_e} \frac{d\omega_a}{d\eta} \bigg|_0 . \quad (III.13)$$

By replacing the value for the constant of integration and solving for $d\omega_a/d\eta$, the final version of Equation (III.12) is:

$$\frac{d\omega_a}{d\eta} = \frac{-V^*}{Nl} + \frac{V^*\omega_a}{Nl} + \frac{\omega_a}{Sa^*} \frac{\rho_0}{Sa_e} \frac{d\omega_a}{d\eta} \Big|_0 . \quad (III.14)$$

The derivative in Equation (III.11) is expanded to get the following.

$$\begin{aligned} \frac{d}{d\eta} (Sa^* (\frac{1}{\omega_a} - 1) (\theta + N4)) \\ = \frac{dSa^*}{d\eta} (\frac{1}{\omega_a} - 1) (\theta + N4) + Sa^* \frac{-1}{\omega_a^2} \frac{d\omega_a}{d\eta} (\theta + N4) \\ + Sa^* (\frac{1}{\omega_a} - 1) \frac{d\theta}{d\eta} \end{aligned} \quad (III.15)$$

The chain rule is applied to $\frac{dSa^*}{d\eta}$.

$$\frac{dSa^*}{d\eta} = \frac{dSa^*}{dSa} \frac{dSa}{dT} \frac{dT}{d\theta} \frac{d\theta}{d\eta} = \left(\frac{T_e - T_\infty}{Sa_e} \right) \frac{dSa}{dT} \frac{d\theta}{d\eta} \quad (III.16)$$

Applying the same procedure to $\frac{dFa^*}{d\eta}$, the result is:

$$\frac{dFa^*}{d\eta} = \left(\frac{T_e - T_\infty}{Fa_e} \right) \frac{dFa}{dT} \frac{d\theta}{d\eta} . \quad (III.17)$$

By combining terms and solving for $\frac{d\theta}{d\eta}$, the final form for equation (III.11) is:

(a) The coefficient of $\frac{d\theta}{d\eta}$ is set equal to $G(\omega_a, \theta)$.

$$G = \frac{N2(T_e - T_\infty)}{Fa_e} \frac{dFa}{dT} + \frac{N3(T_e - T_\infty)}{Sa_e} \frac{dSa}{dT} \left(\frac{1}{\omega_a} - 1 \right) (\theta + N4) + N3(Sa^*) \left(\frac{1}{\omega_a} - 1 \right) . \quad (III.18)$$

(b) Solving for $\frac{d\theta}{d\eta}$, the result is

$$\frac{d\theta}{d\eta} = \left[\frac{N5Sa^*}{\omega_a} + \frac{N3Sa^*}{\omega_a^2} \frac{d\omega_a}{d\eta} (\theta + N4) \right] / G(\omega_a, \theta) \quad (III.19)$$

Equation (III.10) is integrated and the result is:

$$\frac{d\theta}{d\eta} - N7 \int_0^\eta \theta d\eta - N8 H_{fg}^* \frac{Sa^* v^*}{\omega_a} = C . \quad (III.20)$$

Using the boundary condition of $\eta = 0$ where $v^* = 0.0$, the constant of integration is:

$$C = \left. \frac{d\theta}{d\eta} \right|_0 . \quad (III.21)$$

The final version of Equation (III.10) which is solved for v^* is:

$$v^* = \left(\frac{d\theta}{d\eta} \omega_a - N7 \left(\int \theta d\eta \right) \omega_a - \omega_a \left. \frac{d\theta}{d\eta} \right|_0 \right) / (N8 H_{fg}^* Sa^*) . \quad (III.22)$$

The final system of three equations with nondimensional variables is:

(a) Equation (III.14), $\frac{d\omega_a}{d\eta}(V^*, \omega_a, \frac{d\omega_a}{d\eta}\bigg|_0)$:

$$\frac{d\omega_a}{d\eta} = \frac{-V^*}{N1} + \frac{V^*\omega_a}{N1} + \frac{\omega_a}{S_Q^*} \frac{\rho_0}{Sa_e} \frac{d\omega_a}{d\eta}\bigg|_0$$

(b) Equation (III.19), $\frac{d\theta}{d\eta}(\omega_a, \theta, \frac{d\omega_a}{d\eta})$:

$$\frac{d\theta}{d\eta} = \left[\frac{N5Sa^*}{\omega_a} + \frac{N3Sa^*}{\omega_a^2} \frac{d\omega_a}{d\eta} (\theta + N4) \right] / G(\omega_a, \theta)$$

where

$$G(\omega_a, \theta) = \frac{N2(T_e - T_\infty)}{Fa_e} \frac{dFa}{dT} + \frac{N3(T_e - T_\infty)}{Sa_e} \frac{dSa}{dT} \left(\frac{1}{\omega_a} - 1 \right) (\theta + N4) + N3(Sa^*) \left(\frac{1}{\omega_a} - 1 \right)$$

(c) Equation (III.22), $V^*(\omega_a, \int_0^\eta \theta d\eta, \frac{d\theta}{d\eta}, \frac{d\theta}{d\eta}\bigg|_0)$

$$V^* = \left(\frac{d\theta}{d\eta} \omega_a - N7 \left(\int \theta d\eta \right) \omega_a - \omega_a \frac{d\theta}{d\eta}\bigg|_0 \right) / (N8H_{fg}^* Sa^*)$$

The boundary conditions for this set of equations are the following:

(a) For nondimensional temperature:

$$\eta = 0.0 \qquad \theta = \frac{T_c - T_\infty}{T_e - T_\infty} = \theta_c$$

$$\eta = 1.0 \qquad \theta = 1.0$$

$$\left. \frac{d\theta}{d\eta} \right|_0 = \alpha$$

The constant α is not known but, based on laboratory data, can be estimated and through an iteration process the correct value can be obtained. A proposed iteration scheme is presented in Chapter IV.

(b) For working fluid vapor mass fraction:

$$\eta = 0.0 \qquad \omega_a = \omega_{a0}$$

$$\eta = 1.0 \qquad \omega_a = 1.0$$

$$\left. \frac{d\omega_a}{d\eta} \right|_0 = \beta$$

The constant β is not known, but can also be estimated for an initial value and, through an iteration scheme, a correct value can be obtained.

(c) For working fluid vapor dimensionless velocity:

$$\eta = 0.0 \qquad v^* = 0.0$$

E. AUXILIARY EQUATIONS

A means to derive saturated working fluid heat of vaporization, density, and pressure is required before a numerical solution is attempted. Using methanol as the working fluid and data from Eubank [Ref. 8] for saturation conditions, the following functions are derived.

1. Polynomial for the Working Fluid Heat of Vaporization

Tabulated values of the heat of vaporization for methanol are plotted on Figure 3.1 with a solid line. A Fortran program, polyfit, is used to obtain the coefficients of the following polynomial:

$$H_{fg}(T) = aT^2 + bT + c \quad (\text{III.23})$$

where $a = -9.96 \times 10^{-4} \text{ Btu/Lbm}(\text{°R})^2$, $b = .730 \text{ Btu/Lbm}(\text{°R})$, and $c = 397.99 \text{ Btu/Lbm}$. The values of $H_{fg}(T)$ using this polynomial are plotted on Figure 3.1 with a dotted line.

2. Polynomial for the Working Fluid Vapor Saturation Density

Tabulated values for methanol saturated density are plotted on Figure 3.2 with a solid line. Again the Fortran program, polyfit, is used with the following modification:

$$T^* = \frac{T}{T_{\text{ref}}} \quad \text{where} \quad T_{\text{ref}} = 500\text{°R} \quad \text{and} \quad \xi = e^{T^*}$$

A polynomial in the variable ξ is used to describe the saturation density.

SATURATION CH_3OH HFG

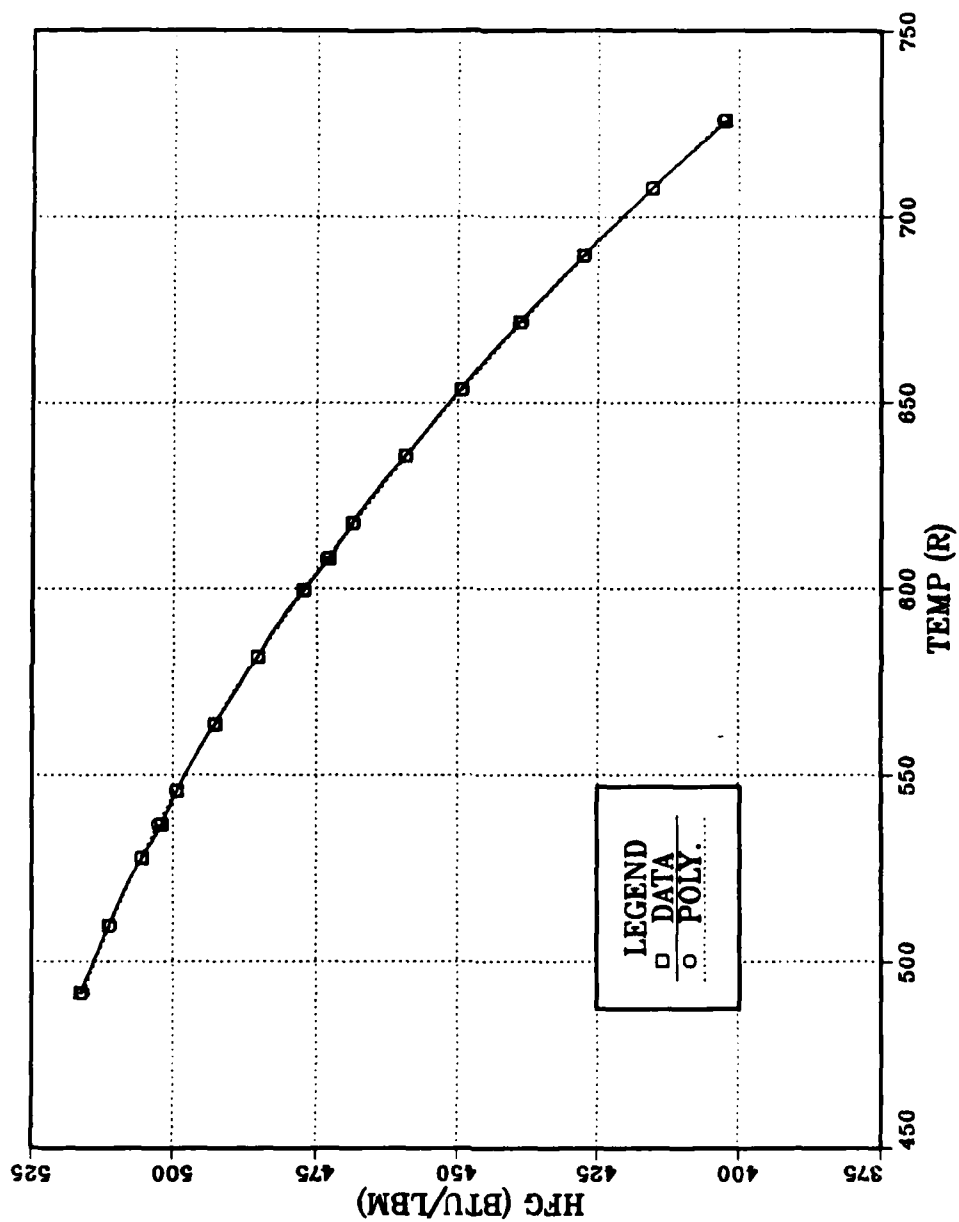


Figure 3.1. Graph of Methanol Heat of Vaporization Versus Temperature

SAT. CH₃OH VAPOR DENSITY

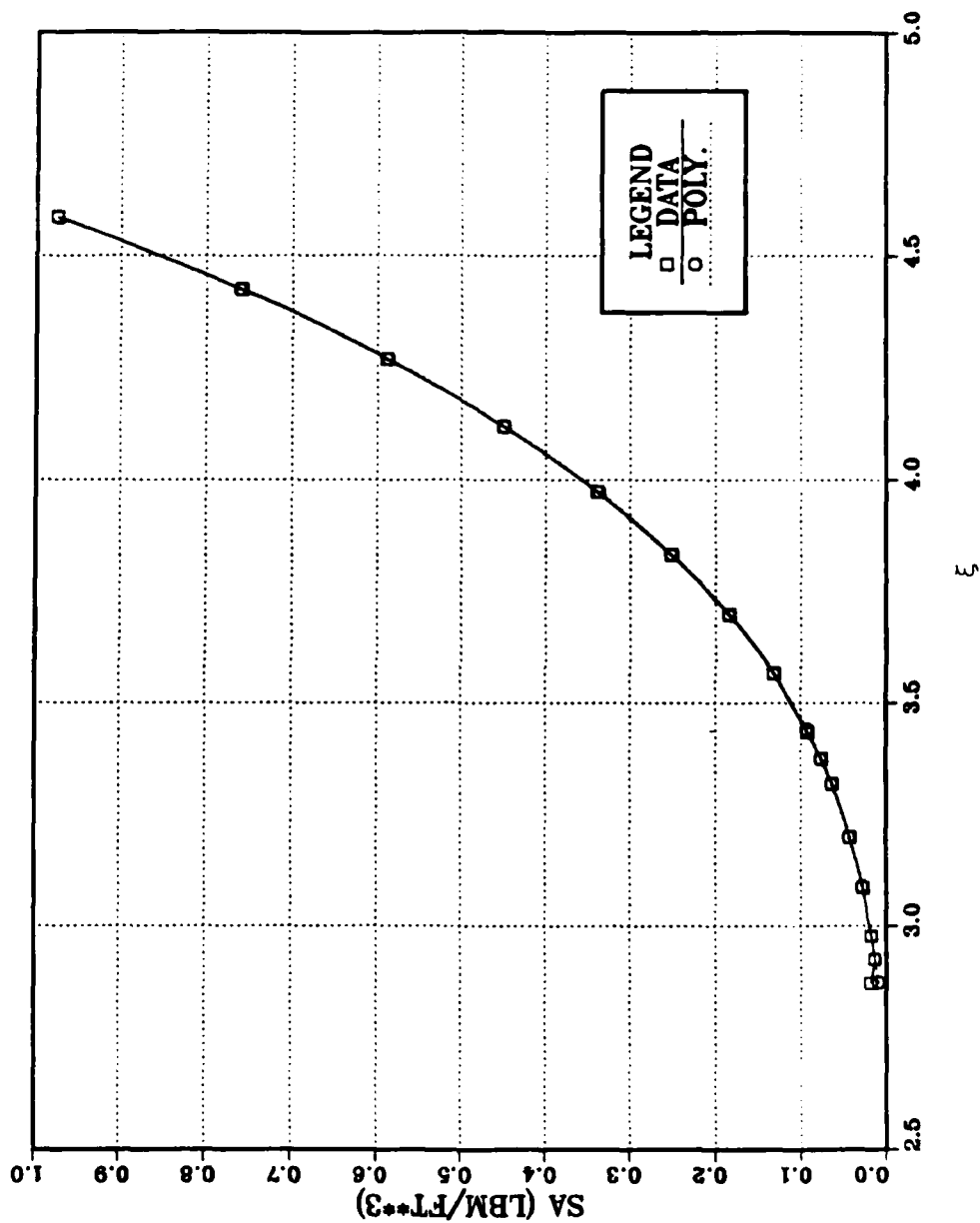


Figure 3.2. Graph of Methanol Saturation Density Versus Nondimensional Temperature

The polynomial, $Sa(\xi)$, is then:

$$Sa(\xi) = a\xi^3 + b\xi^2 + c\xi + d \quad (\text{III.24})$$

where $a = .1293 \text{ Lbm/ft}^3$, $b = -1.0475 \text{ Lbm/ft}^3$, and $c = 2.8868 \text{ Lbm/ft}^3$ and $d = -2.7037 \text{ Lbm/ft}^3$. The value of dSa/dT is also required for the numerical solution. This is:

$$\frac{dSa}{dT} = (3a\xi^3 + 2b\xi^2 + c\xi)/T_{\text{ref}} \quad (\text{III.25})$$

3. Polynomial for the Working Fluid Vapor Saturation Pressure

Tabulated values for methanol saturation pressure are plotted on Figure 3.3 with a solid line. Polyfit is used with the same modification.

The polynomial $Fa(\xi)$ is:

$$Fa(\xi) = a\xi^3 + b\xi^2 + c\xi + d$$

where $a = 24.5994 \text{ Lbf/in}^2$, $b = -191.2472 \text{ Lbf/in}^2$, $c = 498.4940 \text{ Lbf/in}^2$, and $d = -439.9883 \text{ Lbf/in}^2$. The first derivative of $Fa(\xi)$ with respect to temperature is equal to:

$$\frac{dFa}{dT} = (3a\xi^3 + 2b\xi^2 + c\xi)/T_{\text{ref}} \quad (\text{III.26})$$

SAT. CH₃OH VAPOR PRESSURE

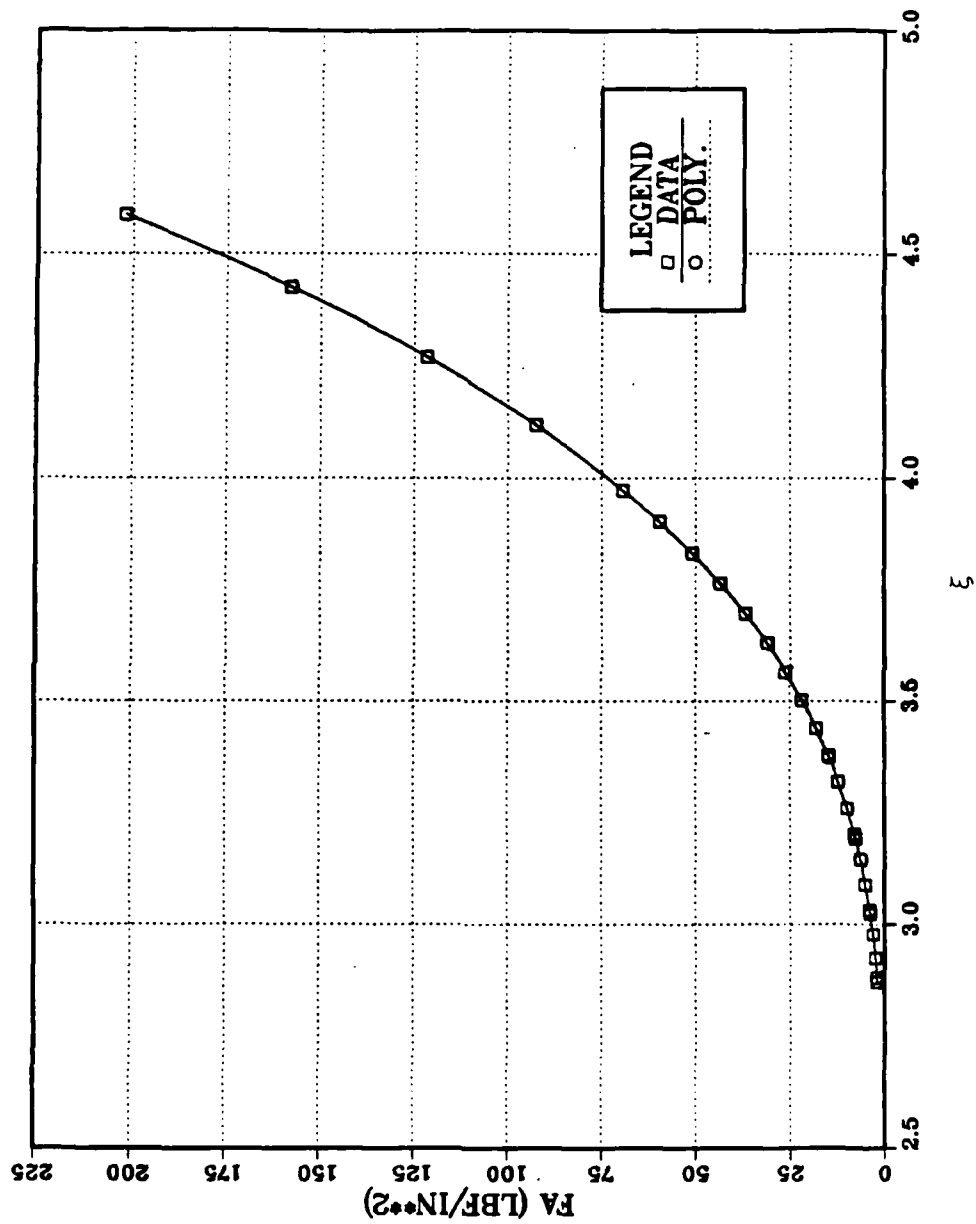


Figure 3.3. Graph of Methanol Saturation Pressure Versus Nondimensional Temperature

IV. DISCUSSION AND RECOMMENDATIONS

A. DISCUSSION

The final version of the model for a variable conductance heat pipe with a vertical orientation is a system of three ordinary, non-linear, coupled, differential equations. These three equations are (III.14), (III.19), and (III.22).

The remaining challenge for the numerical model is to propose an iteration scheme that can solve this system of three equations. A special thank you goes to the collaboration of Dr. M.D. Kelleher and Dr. K.T. Yang for the following iteration technique.

The steps of the iteration procedure are as follows:

(a) To start the iteration process, assume a form for the relationship between θ and ω_a such that $\omega_a = C_1\theta + C_2$. This linear relationship is reasonable because:

- (i) When $\eta = 0$, both ω_a and θ are values close to zero.
- (ii) When $\eta = L$ both ω_a and θ are equal to 1.0.
- (iii) This step is required for the first iteration only.

The equations for this step are then:

$$\omega_a = C_1\theta + C_2 \quad (\text{IV.1})$$

$$\frac{d\omega_a}{d\eta} = C_1 \frac{d\theta}{d\eta} . \quad (\text{IV.2})$$

Then, using Equation (III.19), solve for $\frac{d\theta}{d\eta}$. This establishes a first degree equation, $\frac{d\theta}{d\eta}$, in terms of θ only which can then be integrated numerically to establish a first iteration relationship for $\theta(\eta)$.

(b) With this relationship for θ and $\frac{d\theta}{d\eta}$ defined from step (a) above, Equation (III.22) can be solved for V^* .

(c) Once step (b) is completed, Equation (III.14) can be solved for $d\omega_a/d\eta$. The integral of $d\omega_a/d\eta$ is then used to define ω_a . The process can then be repeated.

In summary, the first iteration goes through steps a-c and the remaining iterations go through steps b-c until there are no further changes in the form of θ , ω_a , and V^* . In theory, the outcome of this numerical iteration is a table which gives dimensionless temperature, θ ; velocity, V^* ; and mass fraction of the working fluid vapor, ω_a , versus non-dimensional condenser length, η .

B. RECOMMENDATIONS

1. This numerical analysis was written to yield results via a computer program. The Continuous System Modeling Program III [Ref. 9] is a computer package that could be used to solve the system of equations. The CSMP [Ref. 9] includes a powerful integral statement which carries out a Fourth Order Runge-Kutta Method.
2. This model assumes constant values for the binary mass diffusion coefficient and the convection heat transfer coefficient. Once numerical values are obtained from the iteration scheme, the accuracy can be improved by using analytical expressions instead of constants for the binary mass diffusion coefficient [Ref. 10] and the convection heat transfer coefficient [Ref. 11].
3. An attempt could be made to model the variable conductance heat pipe that is not in a vertical orientation. Kelleher [Ref. 3] shows that a stratification of the

working fluid and noncondensable gas occurs when the heat pipe is in the horizontal position. This position makes the transport processes inside the pipe three dimensional, which would require a finite difference computer model for solution.

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